a ribbon impeller. For the ribbon pumping downwards, the fluid adjacent to the wall of the tank travels axially downwards with a superimposed angular velocity component. At the bottom it turns inward and then flows upwards along the shaft again with some angular motion. At the surface it turns radially outwards toward the ribbon.

In highly elastic liquids, however, the axial component of velocity was very much suppressed in favor of the angular component. From time to time, the particle was drawn radially into a recirculating vortex and was released again. This phenomenon was observed for other impellers as well and the work was recently summarized by Ulbrecht (1974). Consequences of such a flow pattern with not much intermixing between vortices in a polymerization reactor might be very serious not only for the molecular weight distribution of the polymer produced but also for the potential side reactions.

NOTATION

Ci = circulation number
d_1 = ribbon diameter, Figure 1
d_2 = screw diameter, Figure 1
F = axial thrust, Equation (2)
H = height of the liquid column, Figure 1
h = material parameter, Equation (3)
k = material parameter, Equation (3)
l = length of the impeller, Figure 1
m = material parameter, Equation (3)
N = rotational speed
n = material parameter, Equation (3)
R = radius, Equation (1)
S_1 = pitch of the ribbon, Figure 1
S_2 = pitch of the screw, Figure 1
t = tank diameter
T = torque, Equation (1)
W_1 = width of the ribbon blade
W_2 = width of the screw blade

Greek Letters

α = angle of the conical gap
$\dot{\gamma}$ = shear rate
ω = angular velocity
η = viscosity, Equation (1)
θ_c = circulation time (axial)
θ_c' = circulation time (angular)
θ_m = mixing time
σ_1 = primary normal stress coefficient, Equation (2)

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Magnetofluidynamic Separation of a Binary Gas

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The purpose of this technical note is to show the feasibility of separating the constituents of a gas mixture through magnetofluidynamic means. This may be done when the component that is to be separated has a significantly different ionization potential than the rest of the constituents. Although the analysis presented here may be applied to multicomponent mixtures, the discussion is limited to a binary system. The gas mixture may be preheated to a temperature range where the constituent with the lower ionization potential achieves a certain degree of ionization whereas the other remains un-ionized because of its higher ionization potential. When this mixture is allowed to flow in a magnetic field, the magnetic field acts selectively on the ions and electrons of the ionized constituent. If in addition the field is set oblique to the flow, there should occur a separation of the constituents.

The separation process as envisioned in this system consists of forcing the virgin gas into a constraining magnetic field where, by a process of selective pumping, a net electromagnetically induced drag is exerted on the gas. The drag in turn generates a pressure gradient and thus also a density gradient. Now, since the gas consists of an ionized constituent and a neutral constituent, only the former is constrained by the magnetic field so that when the density gradient is formed, the neutral constituent shall separate from the ionized constituent by diffusion. The amount and degree of separation is dependent on the molecular diffusivity of the binary mixture. The diffusivity is in turn dependent on the temperature and pressure.

The electromagnetically induced drag is generated when the electrons and ions are forced to cross the magnetic field via the pumping process, thereby causing a motional emf $\mathbf{U}_{elec} \times \mathbf{B}$ to appear. In turn, a current $\mathbf{J} = \sigma \mathbf{U}_{elec} \times \mathbf{B}$ is generated producing a force $\mathbf{J} \times \mathbf{B}$ that acts primarily on the electrons and opposes their flow across the field lines (Cowling, 1957). The force $\mathbf{J} \times \mathbf{B}$ is the drag that is induced on the gas and is transmitted through collisions with ions and neutral atoms. To achieve a desired amount of separation in a steady state system, the procedure would be to provide for a sufficiently large area for diffusion and at the same time maintain the required pressure gradient that causes the diffusion.

PHYSICAL MODEL AND GOVERNING EQUATIONS

The physical model and coordinate system is shown in Figure 1. It is assumed that a single charge ion is formed through the ionic equation.

$$\mathcal{A} \rightleftarrows \mathcal{A}^+ + e^- \tag{1}$$

Denoting the inert constituent by $\mathcal{B}$, the gas shall then have four species $\mathcal{A}$, $\mathcal{A}^+$, $\mathcal{B}$, and the free electrons. Strict two-dimensionality is imposed so that the velocity, current density, magnetic flux, and electric field vectors assume the forms

$$\begin{aligned} \mathbf{V} &= \mathbf{V}(V_x, V_y, 0) \\ \mathbf{J} &= \mathbf{J}(0, 0, J_z) \\ \mathbf{B} &= \mathbf{B}(B_x, B_y, 0) \\ \mathbf{E} &= \mathbf{E}(0, 0, E_z) \end{aligned} \tag{2}$$

In order to show the process of separation, the governing equations are derived for simple conditions. (A more complete presentation of the equations are given in a supplement to this technical note.)* To begin with, it is assumed that thermal and ionization equilibrium exists in the plasma. Under these conditions, the degree of ionization of the ionizing constituent can be predicted through the Saha equation. The electrical conductivity of the gas is dependent on the degree of ionization. However, when Coulomb interactions dominate, the conductivity of the gas becomes relatively insensitive to the degree of ionization and instead becomes strongly sensitive to the temperature via the 3/2 power law (Rosa, 1968). For this to occur, the degree of ionization need only be a few percent.

As an example, the undisturbed magnetic field is assumed to be of the form

$$B_x = B_0\, e^{\beta x}, \quad B_y = -\beta\, B_0\, e^{\beta x} y \quad x \leq L$$

The magnetic mirror ratio β is assumed sufficiently small so that the axial velocity V_x can be assumed much greater than the normal velocity V_y. Upon neglect of streamwise derivatives in the diffusion and momentum equations and neglect of second-order disturbances in the magnetic field, the governing equations for the system become (Hirschfelder et al., 1954):

Mass conservation

$$\frac{\partial}{\partial \chi}(\rho V_x) + \frac{\partial}{\partial y}(\rho V_y) = 0 \tag{3}$$

Species conservation

$$\rho V_x \frac{\partial w_B}{\partial \chi} + \rho V_y \frac{\partial w_B}{\partial y} = -\frac{\partial}{\partial y}(\rho_B u_B) \tag{4}$$

$$\begin{gathered} \rho_B u_B + \rho_A u_A = 0 \\ w_A + w_B = 1 \end{gathered} \tag{5}$$

where w_B is the mass fraction of the neutral species $\mathcal{B}$ and w_A is the sum of the mass fractions of the electrons, ions, and un-ionized molecules of $\mathcal{A}$; u_A and u_B are respectively the diffusive velocities of species $\mathcal{A}$ and $\mathcal{B}$.

Momentum conservation

$$\rho V_x \frac{\partial V_x}{\partial \chi} + \rho V_y \frac{\partial V_x}{\partial y} = -\frac{\partial p}{\partial \chi} - J_z\, B_y + \frac{\partial}{\partial y}\left(\mu \frac{\partial u}{\partial y}\right) \tag{6}$$

$$\frac{\partial p}{\partial y} \simeq J_z\, B_x \tag{7}$$

$$\frac{w_B w_A}{D_{AB}}(u_A - u_B) = \frac{\partial w_B}{\partial y} + (\omega_B - w_B)\frac{\partial \ln p}{\partial y} + \frac{w_B}{p} J_z\, B_x \tag{8}$$

Ohm's law

$$J_z = \sigma(V_x\, B_y - V_y\, B_x) \tag{9}$$

Energy conservation

$$\rho V_x \frac{\partial H_T}{\partial \chi} + \rho V_y \frac{\partial H_T}{\partial y} = \frac{\partial}{\partial y}(\dot{q}_{cy} + \dot{q}_{Ry}) + \Phi \tag{10}$$

Equations (3) to (10) form a parabolic system of partial differential equations. The boundary conditions for such a system shall then be the specification of the profiles of velocity, temperature, concentration, and pressure at the inlet and conditions at specified values of the normal coordinate y. The inlet profiles for pressure and axial velocity were arbitrarily chosen to be Gaussian in form. The temperature at the inlet was assumed constant. Conditions at $y \to \infty$ and $y = 0$ were as follows:

$$y \to \infty \qquad u = 0, \quad p = p_\infty, \quad T = T_\infty$$

$$y = 0 \qquad \frac{\partial u}{\partial y} = 0 \quad \frac{\partial T}{\partial y} = 0 \quad \frac{\partial p}{\partial y} = 0$$

The simplified system of equations were numerically solved using the second-order accurate Crank-Nicholson implicit finite difference scheme.

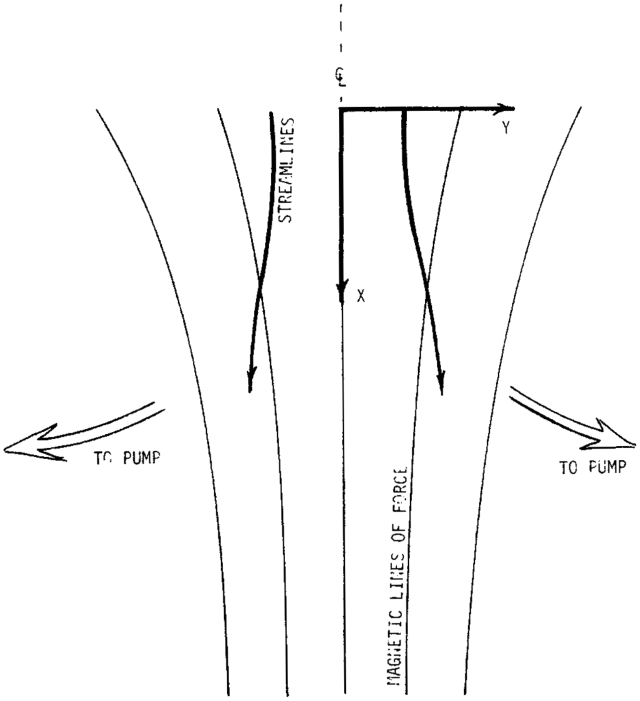

Fig. 1. Physical model and coordinate system.

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* The Supplement has been deposited as Document No. 02593 with the National Auxiliary Publications Service (NAPS), c/o Microfiche Publications, 440 Park Ave. So., N.Y., N.Y. 10016 and may be obtained for \$1.50 for microfiche or \$5.00 for photocopies.

RESULTS AND DISCUSSION

By way of example, the calculations were applied to a mixture of 90% aluminum (ionizable constituent) and 10% carbon monoxide (neutral constituent) (Chang et al., 1972). The temperature was 5000°K while the operating pressure was assumed to be a tenth of an atmosphere. A magnetic flux density of six teslas was used and calculations were carried out to the point of maximum flux density. The mass fraction distribution of the neutral species were plotted as a function of distance from the inlet and shown in Figure 2. As expected, a depletion of the neutral species occurs towards the center and very noticeable enrichment away from the center. This comes about as a result of the constraining effect of the magnetic field on the ionizing species. The separation is dependent on the rate of diffusion of the neutral species away from the center. The diffusion rate is in turn determined by the steepness of the pressure gradient generated by the flow as seen in Figure 3. Plots of the bulk velocity, temperature, and mass flux distribution of the neutral species are shown in Figures 4 to 6. The degree of separation is very much dependent on the steepness of the pressure gradient that causes the diffusion and the cross-sectional area through which the diffusion is allowed to take place. The pressure gradient is governed by the conductivity of the plasma, the strength of the magnetic field, and the velocity of the gas. The limitations of such a system are of course inherent in these three main parameters. Present day magnet technology makes the magnetic fields of about ten teslas feasible (Rosa, 1968). This being the case, a high degree of separation is possible.

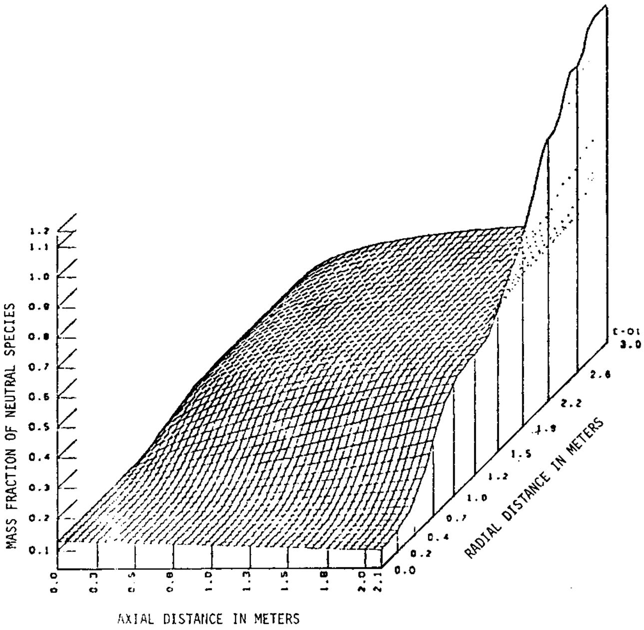

Fig. 2. Mass fraction distribution of neutral species.

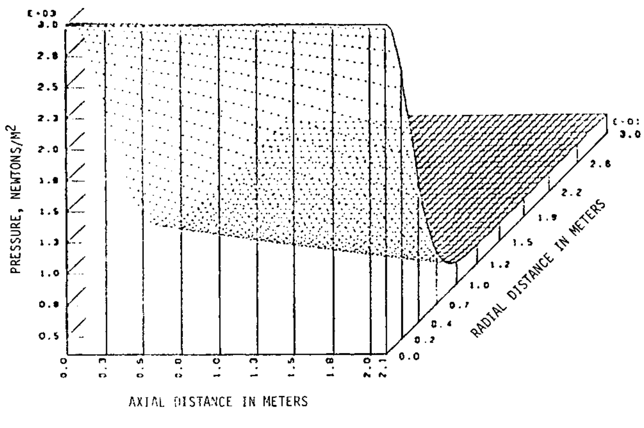

Fig. 3. Pressure distribution.

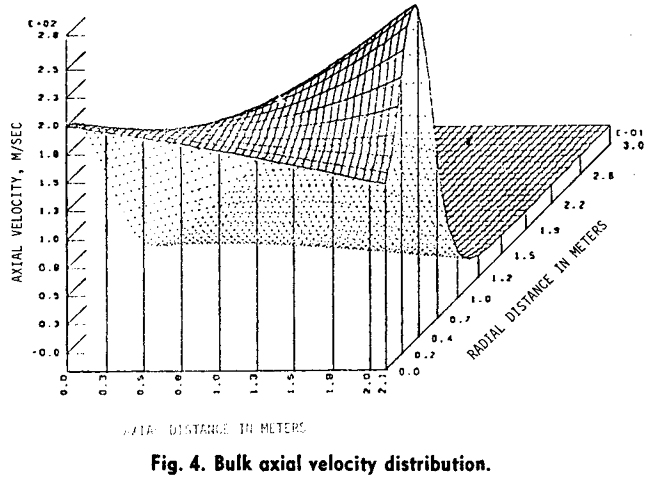

Fig. 4. Bulk axial velocity distribution.

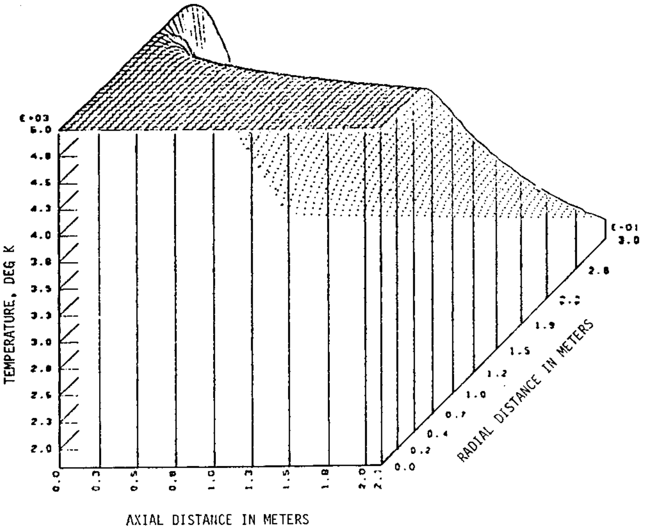

Fig. 5. Temperature distribution.

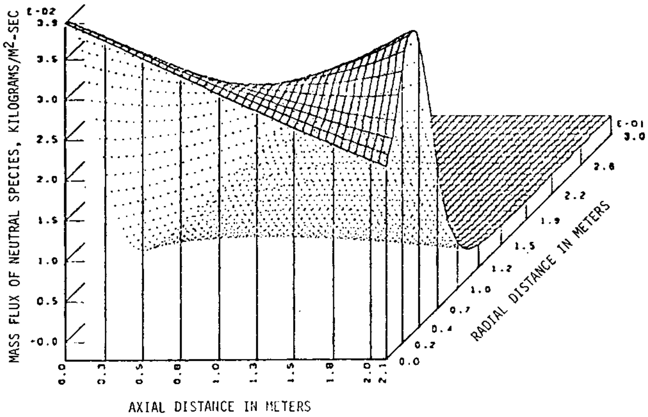

Fig. 6. Mass flux distribution of neutral species.

NOTATION

B_x, B_y = magnetic flux density components
$\mathbf{B}$ = magnetic flux vector
D_{AB} = binary diffusion coefficient
H_T = total enthalpy
$\mathbf{J}$ = current density vector
M_x, M_y = mass flux components
N_A, N_B = number density of ionized and neutral constituents respectively
p = total pressure
$(V_x = U_1, V_y = v)$ = bulk velocity components
u_A, u_B = diffusive velocities of ionized and neutral constituents respectively
w_A, w_B = mass fractions of ionized and neutral species respectively
q_c, q_r = conductive and radiative fluxes
ρ = mass density
ω = mole fraction
Φ = dissipative energy source term
σ = gas conductivity
μ = viscosity

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The Prediction of Azeotropic Behavior and Saturated Liquid Densities in the Carbon Dioxide-Ethane System

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Gugnoni et al. (1974) have recently discussed the applicability of the Benedict-Webb-Rubin equation to the calculation of vapor-liquid equilibria, including azeotropic behavior, in the carbon dioxide-ethane system. In order to correlate their data, they modified the mixing rule for one of the constants of the equation of state by means of a temperature-dependent binary interaction coefficient. A similar temperature dependent coefficient was required by Hamam and Lu (1974) for correlating carbon dioxide-ethane vapor-liquid equilibria using a modified Redlich-Kwong equation. Thus the equation of state method is not simple to use, especially for predictive purposes, as the interaction coefficient must first be obtained as a function of temperature for each particular case.

The purpose of this note is to alert readers to another technique, described in detail elsewhere, which can be used to correlate and predict the behavior of the carbon dioxide-ethane system. This technique makes use of the extended form of the corresponding states principle as described by Rowlinson and Watson (1969). The method has been used successfully for the calculation of vapor-liquid equilibria (Watson and Rowlinson, 1969; Gunning and Rowlinson, 1973) and critical and azeotropic states (Teja and Rowlinson, 1973, Teja and Kropholler, 1975). Here, after a brief outline of the method, predicted azeotropic states and densities in the carbon dioxide-ethane system are presented and compared with the experimental results of Gugnoni et al. (1974).

CORRESPONDING STATES METHOD

Two pure substances α and 0 are defined to be in corresponding states if the configurational Helmholtz free energy of substance α at density ρ and temperature T may be obtained from the configurational Helmholtz free energy of the reference substance 0 at density $\rho h_{\alpha\alpha,0}$ and temperature $T/f_{\alpha\alpha,0}$ as follows: (The subscripts $\alpha\alpha,0$ signify a property of α relative to substance 0).

$$A_\alpha[\rho, T] = f_{\alpha\alpha,0}\, A_0[\rho h_{\alpha\alpha,0}, T/f_{\alpha\alpha,0}] - RT \ln h_{\alpha\alpha,0} \qquad (1)$$

If an analytic expression for the configurational Helm-